The Basics of Terminological Logics

- Predicate Logic
- Thue Systems
- Polymodal Logic
- Model Theory
- Regular Languages
- Dynamic Logic
- Constraint Solving
- Feature Logic
- Terminological Logic
- KL-ONE languages
  - Feature Logic
    - set-valued features
    - functional uncertainty
  - Complex Objects
    - subtype inference
    - satisfiability

adapted from B. Nebel
Knowledge Representation with Terminological Logics

Terminological Logic = Concept Language = Term Subsumption Language = Description Logics = KL-ONE (Knowledge Language) Derivatives

Main Goal: Representation of *terminological knowledge* based on *structured inheritance networks*

Concepts and Taxonomies:

- ‘Concept’ as atomic unit of knowledge represents a class of entities of a discourse domain
- Concept is: ■ named after the elements of the class
  ■ graphically illustrated as ellipse, that is framing its name
- A concept, that represents a set S, is denoted as |C|S
- If S ⊂ T, then: |C|S subconcept (Sub C) of |C|T
  |C|T super concept (SuperC) of |C|S
- This relation is: ■ called *concept subsumption* and
  ■ displayed as bold, directed arrow from |C|S to |C|T
- You say: |C|T *subsumes* |C|S
  |C|S *specializes* |C|T
Concepts and Taxonomies

Concept Subsumption

- is transitive:
  If $|C|A$ subsumes $|C|B$ and $|C|B$ subsumes $|C|C$, then $|C|A$ subsumes $|C|C$.

- is antisymmetric:
  If $|C|A$ subsumes $|C|A'$ und $|C|A'$ subsumes $|C|A$, then $|C|A = |C|A'$

- Induces a partial order on a set of concepts and yields graphically to a directional acyclic graph, whose edges correspond to subsumption relations.

Each network contains the most general super concept THING as a root which always subsumes all other concepts of a network autarkically.

If a new concept should be included into an existing network, it must be placed at the correct position within in the taxonomy.

**Rule:**

The new concept $|C|A$ should subsume all concepts, which are less general than $|C|A$ and it should specialize all concepts, which are less specific than $|C|A$

This process of incorporating a new concept according to a rule is referred to as classification, and is performed by a so called classifier.

The order of incremental updates of a cycle-free network does not influence the structure of the net.
Subsumption relation in NIKL

Note: MICRO-COMPUTER specializes DIGITAL-COMPUTER and ELECTRICAL-DEVICE.
RADIO is not a VEGETABLE, since this subsumption is not directly represented in the network and cannot be derived.
Primitive and Disjoint Classes

- A concept marked as **primitive class** (graphically tagged with \( \square \)) specifies **necessary**, but **not sufficient** conditions for the affiliation to the class, it represents.

- Other concepts can only be subsumed by a concept that is marked as a primitive class, if this is explicitly stated in their definition which means that the classifier does not create subsumption relations between new concepts and primitive classes.

- You can combine concepts to a **disjoint class** (graphically: a connection line that 'bridges' subsumption relations) to express, that these concepts represent disjoint sets.
  The membership in a disjoint class is a transitive attribute, i.e. concepts that are subsumed by different **branches** of a disjoint class, are also disjoint.
Covering and Partitions

- A concept $|C|A$ is **covered** (graphically: ’connected circles‘ on subsumption arrows) by a set of concepts $X$, if each instance of $|C|A$ is also an instance of at least one concept $X$.

- Each computer must be in one of the four classes, but it can also specialize several of them.

- A **partition** is an covering, which is also a disjoint class at the same time.

- It is a union of mutually disjoint sets, whose combination is a superset of the partitioned set.

- An electric charge can only have exactly one attribute: positive, negative or neutral
Role and Role Filler

- A **role** defines the relation between two concepts.
  A role is a binary relation and is graphically illustrated as a box in a circle.
  An undirected edge is drawn from the **domain** of the role to the role symbol.
  From the role symbol a directed edge leads to the **range** of the role.

- An instance of the range of a role is called **role filler**.

- 'All inhabitants of an inhabited planet are life forms.'
  \[ \forall x \in \text{INHABITED-PLANET}(x) \ \exists \ y \in \text{LIFE-FORM}(y) \land \text{INHABITANT}(x,y) \]

- The role \(|R| \ \text{INHABITANT}\) represents a relation, which is inherited to all subconcepts of the domain.

- A general method to define subconcepts is the **range restriction** of the inherited role.
Role Taxonomy

- If a role $|R|A$ represents a more specific relation than $|R|B$, then we call $|R|A$ subrole (SubR) of $|R|B$ and $|R|B$ super role (SuperR) of $|R|A$.

- $|R|A$ differentiates $|R|B$ and $|R|B$ generalizes $|R|A$.

- On top of this taxonomy is the MostGeneralRole. Regarding this super role as a concept (this is possible for all roles), leads to the following:

- Primitive roles are roles that differentiate their super roles in a way that is inaccessible for the system.
Inverse Roles

• Another possibility to differentiate a role is to declare it as an **inverse role** of some other role. The inverse role is inherited to all subroles. For each instance/filler pair of a role their is a corresponding instance/filler pair of the inverse role.

• The relationship between a role and a Ist inverse role is indicated graphically by a bidirectional edge.

  ![Graphical Representation](image.png)

  - A government rules a country and a country is ruled by a government.
  - With inverse roles, the domain and range is interchanged.
  - Some classifiers do not consider inverse roles.
Role Restrictions

• A role restriction consists of two components:
  – a value restriction (vr)
  – a number restriction (nr)

• The value restriction constraints the number of possible role fillers.
• The number restriction defines the lower und upper bounds of the cardinality of the set of role fillers.

0 := no lower bound
∞ (or NIL) := no upper bound
(u o) := u is the lower bound o is the upper bound, u < o; u, o ≥ 0
(k k) = (k) := exactly k elements.

∀ x ∈ JET-PLANE(x) ∃ y ∈ JET-ENGINE(y) ∧ POWERED-BY(x,y)
Role Value Maps

• To define a concept exactly, it is often necessary to define restrictions not only for single roles, but also to postulate relationships between the sets of role fillers of two roles with regard to one concept.

• This is achieved by defining role value maps (or role constraints), that consist of two role chains and a constraint type (subset, superset, identity).

• A role chain is a list of roles where the first element is linked to the concept, for which the role value map is defined.

• Since every role in a role chain corresponds to a relation, the role chain leads to a composition of those relations. The domain of the composed relation is the concept for which the role value map is defined.

• Thus, with role value maps we can represent relations that ’span‘ various concepts, roles and levels of taxonomy.

• The constraint type is represented graphically by a rhombus that contains the symbol of the corresponding relationship of sets.

• Dotted lines that are connected to the rhombus represent the role chain.
An Example for Role Value Maps

- In the following example the mother tongue of a person is defined as the language that the residents of that city speak, in which the person was born.

1. Role chain: MOTHER-TONGUE (PERSON) =

2. Role chain: SPEAK (RESIDENTS (LOCATION (BIRTH (PERSON))))

Representation without copying the node PERSON:
An Example for the Definition of a Concept: The Hydrogen Atom
Definition of the Hydrogen Atom

(DEFCONCEPT UNIT-OF-MATTER PRIMITIVE (SPECIALIZES THING))
(DEFCONCEPT PARTICLE PRIMITIVE (SPECIALIZES UNIT-OF-MATTER))
(DEFCONCEPT NEUTRON PRIMITIVE (SPECIALIZES PARTICLE))
(DEFCONCEPT CHARGED-PARTICLE PRIMITIVE (SPECIALIZES PARTICLE))
(DEFCONCEPT ELECTRON PRIMITIVE (SPECIALIZES CHARGED-PARTICLE))
(DEFCONCEPT PROTON PRIMITIVE (SPECIALIZES CHARGED-PARTICLE))
(DEFCONCEPT CHARGE-DICHOTOMY (ELECTRON PROTON))
(DEFCONCEPT ATOM-OR-ION PRIMITIVE (SPECIALIZES UNIT-OF-MATTER))
  (ROLE CONTAINS-1  (VRCONCEPT NEUTRON)
    (MIN 0) (MAX NIL))
  (ROLE CONTAINS-2  (VRCONCEPT CHARGED-PARTICLE)
    (MIN 1) (MAX NIL)))
(DEFCONCEPT HYDROGEN-ATOM PRIMITIVE (SPECIALIZES ATOM-OR-ION))
  (ROLE CONTAINS-3  (DIFFERENTIATES CONTAINS-2)
    (VRCONCEPT ELECTRON (NUMBER 1)))
  (ROLE CONTAINS-4  (DIFFERENTIATES CONTAINS-2)
    (VRCONCEPT PROTON (NUMBER 1)))
Definition of a Circle as a Subconcept of an Ellipse

(DEFCONCEPT MATHEMATICAL-OBJECT (SPECIALIZES THING))
(DEFCONCEPT LINE-SEGMENT
  PRIMITIVE (SPECIALIZES MATHEMATICAL-OBJECT))
(DEFCONCEPT CLOSED-CURVE
  PRIMITIVE (SPECIALIZES MATHEMATICAL-OBJECT))
(DEFCONCEPT ELLIPSE
  PRIMITIVE (SPECIALIZES CLOSED-CURVE)
  (ROLE MINOR-AXIS (VRCONCEPT LINE-SEGMENT) (NUMBER 1))
  (ROLE MAJOR-AXIS (VRCONCEPT LINE-SEGMENT) (NUMBER 1)))
(DEFCONCEPT CIRCLE (SPECIALIZES ELLIPSE) ( = (MINOR-AXIS) (MAJOR-AXIS)))
Automatic Classification

(DEFCONCEPT GRANDFATHER
 (SPECIALIZES MAN)
 (ROLE CHILD (VR PARENT) (MIN 1)))

(DEFCONCEPT PARENT
 (SPECIALIZES HUMAN)
 (ROLE CHILD (MIN 1)))

SPECIALIZES-Edge

HUMAN

PARENT

MAN

GRANDFATHER

SPECIALIZES-Edge

INCOME

MONEY

HUMAN

PARENT

GRANDCHILD

CHILD

CHILD

CHILD

CHILD

CHILD

CHIL
Automatic Classification

(DEFCONCEPT GRANDFATHER
(SPECIALIZES MAN)
(ROLE CHILD (VR PARENT) (MIN 1)))

(DEFCONCEPT PARENT
(SPECIALIZES HUMAN)
(ROLE CHILD (MIN 1)))

SPECIALIZES-Edge calculated subsumption-relation
Inferential Services in Terminological Logics

**In the T-Box:**

Are all concepts different or are there equivalences? (Equivalence)
Which subconcept relationships exist? (Subsumption/Classification)
Which role relationships apply therefore to those concepts? (Inheritance)
Is the defined concept meaningful or is the class categorically empty? (Incoherence)
Do two classes share instances or not? (Disjunctiveness)

**In the Complete Knowledge-Base (T-Box and A-Box):**

Is the KB consistent? (Consistency)
Which classes instantiate a given object? (Instantiation/Realization)
Which known elements contains a given class? (Retrieval)
Which (role-)relationships consist between various objects? (Inheritance)
Inferential Services

T-Box:

Subsumption: $\forall x \text{ Truck}(x) \Rightarrow \text{Transport Vehicle}(x)$?

Inheritance: $\forall x \text{ Mother\_without\_Sons}(x) \Rightarrow \exists y \text{ Children}(x,y)$

Incoherence: $\forall x \neg \text{ Grandfather}(x)$? or: ex. Interpretation $\mathcal{I} \not\models \exists x \text{ Grandfather}(x)$?

Disjunctness: $\neg \exists x \text{ Father}(x) \land \text{Mother}(x)$?

Knowledge Base (KB):

Consistency: does a model of the KB exist, i.e. ex. Interpretation $\mathcal{I} \models \text{KB}$?

Instantiation/Realization: Goodstrain(Z#521)?

respectively find all concepts C mit C(Z#521)!

Retrieval: Find all objects c with woman(c)!

Inheritance: Children(Karin,Eva)?
Formal Semantics of Terminological Logics

Why?

- Specification of the meaning of representation constructs
- Comparability with other formalisms
- Algorithmization
- Correctness and complexity of inference procedures

How?

- Concepts refer to a set of instances: *Concept extension*
- Roles describe relations between those instances: *Role extension*
- Concept descriptions specify *necessary* and *sufficient* conditions for instances
- *Subsumption* is the *necessary inclusion* of concept extensions
- *Inconsistency* is the *necessary emptyness* of concept extensions
A Model-Theoretic Semantics for Description Logics

Interpretation function

Interpretation domain

Concepts
- Lawyer
- Doctor
- Vehicle

Roles
- hasChild
- owns

(Lawyer \cap Doctor)