The Basics of Terminological Logics

Concepts and Taxonomies

Knowledge Representation with Terminological Logics

Subsumption relation in NIKL

Note: MICRO-COMPUTER specializes DIGITAL-COMPUTER and ELECTRICAL-DEVICE. RADIO is not a VEGETABLE, since this subsumption is not directly represented in the network and cannot be derived.
Primitive and Disjoint Classes

- A concept marked as **primitive class** (graphically tagged with \(\square\)) specifies **necessary**, but not **sufficient** conditions for the affiliation to the class, it represents.
- Other concepts can only be subsumed by a concept that is marked as a primitive class, if this is explicitly stated in their definition which means that the classifier does not create subsumption relations between new concepts and primitive classes.
- You can combine concepts to a **disjoint class** (graphically: a connection line that ‘bridges’ subsumption relations) to express, that these concepts represent disjoint sets.

The membership in a disjoint class is a transitive attribute, i.e. concepts that are subsumed by different **branches** of a disjoint class, are also disjoint.

Covering and Partitions

- A concept \(|C|A\) is **covered** (graphically: ‘connected circles’ on subsumption arrows) by a set of concepts \(X\), if each instance of \(|C|A\) is also an instance of at least one concept \(X\).
- Each computer must be in one of the four classes, but it can also specialize several of them.
- A **partition** is an covering, which is also a disjoint class at the same time.
- It is a union of mutually disjoint sets, whose combination is a superset of the partitioned set.
- An electric charge can only have exactly one attribute: **positive**, **negative** or **neutral**

Role and Role Filler

- A **role** defines the relation between two concepts.

  A role is a binary relation and is graphically illustrated as a box in a circle.

  An undirected edge is drawn from the **domain** of the role to the role symbol.

  From the role symbol a directed edge leads to the **range** of the role.

- An instance of the range of a role is called **role filler**.

Role Taxonomy

- If a role \(|R|A\) represents a more specific relation than \(|R|B\), then we call \(|R|A\) **subrole** (SubR) of \(|R|B\) and \(|R|B\) **super role** (SuperR) of \(|R|A\).
- \(|R|A\) **differentiates** \(|R|B\) and \(|R|B\) **generalizes** \(|R|A\).

  - On top of this taxonomy is the MostGeneralRole. Regarding this super role as a concept (this is possible for all roles), leads to the following:
    - Primitive roles are roles that differentiate their super roles in a way that is inaccessible for the system.
**Inverse Roles**

- Another possibility to differentiate a role is to declare it as an **inverse role** of some other role. The inverse role is inherited to all subroles.
- For each instance/filler pair of a role their is a corresponding instance/filler pair of the inverse role.
- The relationship between a role and a 1st inverse role is indicated graphically by a bidirectional edge.

![Diagram of inverse roles](image)

- A government rules a country and a country is ruled by a government.
- With inverse roles the domain and range is interchanged.
- Some classifiers do not consider inverse roles.

**Role Restrictions**

- A role restriction consists of two components:
  - a **value restriction (vr)**
  - a **number restriction (nr)**
- The value restriction constrains the number of possible role fillers.
- The number restriction defines the lower und upper bounds of the cardinality of the set of role fillers.

\[
0 := \text{no lower bound} \\
\infty \text{ (or NIL)} := \text{no upper bound} \\
(u, o) := u \text{ is the lower bound o is the upper bound, } u < o; \ u, o \geq 0 \\
(k, k) = (k) := \text{exactly } k \text{ elements.}
\]

\[
\forall x \in \text{JET-PLANE}(x) \exists y \in \text{JET-ENGINE}(y) \land \text{POWERED-BY}(x, y)
\]

**Role Value Maps**

- To define a concept exactly, it is often necessary to define restrictions not only for single roles, but also to postulate **relationships between the sets of role fillers of two roles** with regard to one concept.
- This is achieved by defining **role value maps** (or role constraints), that consist of two role chains and a **constraint type** (subset, superset, identity).
- A **role chain** is a list of roles where the first element is linked to the concept, for which the role value map is defined.
- Since every role in a role chain corresponds to a relation, the role chain leads to a **composition** of those relations. The domain of the composed relation is the concept for which the role value map is defined.
- Thus, with role value maps we can represent relations that span various concepts, roles and levels of taxonomy.
- The constraint type is represented graphically by a rhombus that contains the symbol of the corresponding relationship of sets.
- Dotted lines that are connected to the rhombus represent the role chain.

**An Example for Role Value Maps**

- In the following example the mother tongue of a person is defined as the language that the residents of that city speak, in which the person was born.

1. Role chain: MOTHER-TONGUE (PERSON) =
2. Role chain: SPEAK (RESIDENTS (LOCATION (BIRTH (PERSON))))

Representation without copying the node PERSON:
An Example for the Definition of a Concept: The Hydrogen Atom

**Definition of the Hydrogen Atom**
- (DEFCONCEPT UNIT-OF-MATTER PRIMITIVE (SPECIALIZES THING))
- (DEFCONCEPT PARTICLE PRIMITIVE (SPECIALIZES UNIT-OF-MATTER))
- (DEFCONCEPT NEUTRON PRIMITIVE (SPECIALIZES PARTICLE))
- (DEFCONCEPT CHARGED-PARTICLE PRIMITIVE (SPECIALIZES PARTICLE))
- (DEFCONCEPT ELECTRON PRIMITIVE (SPECIALIZES CHARGED-PARTICLE))
- (DEFCONCEPT PROTON PRIMITIVE (SPECIALIZES CHARGED-PARTICLE))
- (DEFCONCEPT CHARGE-DICHOTOMY (ELECTRON PROTON))
- (DEFCONCEPT HYDROGEN-ATOM PRIMITIVE (SPECIALIZES UNIT-OF-MATTER)
  - (ROLE CONTAINS-1 (VRCONCEPT NEUTRON) (NUMBER 1))
  - (ROLE CONTAINS-2 (VRCONCEPT CHARGED-PARTICLE) (NUMBER 1))
  - (ROLE CONTAINS-3 (DIFFERENTIATES CONTAINS-2) (VRCONCEPT ELECTRON) (NUMBER 1))
  - (ROLE CONTAINS-4 (DIFFERENTIATES CONTAINS-2) (VRCONCEPT PROTON) (NUMBER 1)))

**Definition of a Circle as a Subconcept of an Ellipse**

- (DEFCONCEPT MATHEMATICAL-OBJECT (SPECIALIZES THING))
- (DEFCONCEPT LINE-SEGMENT PRIMITIVE (SPECIALIZES MATHEMATICAL-OBJECT))
- (DEFCONCEPT CLOSED-CURVE PRIMITIVE (SPECIALIZES MATHEMATICAL-OBJECT))
- (DEFCONCEPT ELLIPSE PRIMITIVE (SPECIALIZES CLOSED-CURVE)
  - (ROLE MINOR-AXIS (VRCONCEPT LINE-SEGMENT) (NUMBER 1))
  - (ROLE MAJOR-AXIS (VRCONCEPT LINE-SEGMENT) (NUMBER 1)))
- (DEFCONCEPT CIRCLE (SPECIALIZES ELLIPSE) (= (MINOR-AXIS) (MAJOR-AXIS)))

**Automatic Classification**
- (DEFCONCEPT GRANDFATHER (SPECIALIZES MAN)
  - (ROLE CHILD (VR PARENT) (MIN 1)))
- (DEFCONCEPT PARENT (SPECIALIZES HUMAN)
  - (ROLE CHILD (VR PARENT) (MIN 1)))

- (DEFCONCEPT INCOME
  - (VRGR PARENT) (MIN 1))
- (DEFCONCEPT MONEY (SPECIALIZES-Edge)
  - (S PARENT) (VRP PARENT) (MIN 1))

- (DEFCONCEPT MAN
  - (VR PARENT) (MIN 1))
- (DEFCONCEPT GRANDCHILD
  - (VR PARENT) (MIN 1))

- (DEFCONCEPT HUMAN
  - (VR PARENT) (MIN 1))
- (DEFCONCEPT GRANDFATHER
  - (VR PARENT) (MIN 1))
- (DEFCONCEPT PARENT
  - (VR PARENT) (MIN 1))
- (DEFCONCEPT GRANDCHILD
  - (VR PARENT) (MIN 1))
- (DEFCONCEPT HUMAN
  - (VRP PARENT) (MIN 1))
- (DEFCONCEPT SPECIALIZES-Edge
  - (S PARENT))
**Automatic Classification**

(DEFCONCEPT GRANDFATHER
(SPECIALIZES MAN)
(ROLE CHILD (VR PARENT) (MIN 1)))

- HUMAN
  - PARENT
    - CHILD
      - GRANDFATHER
        - GRANDCHILD
          - MONEY
  - MAN
  - HUMAN

**Inferential Services**

**T-Box:**

Subsumption: $\forall x \text{ Truck}(x) \rightarrow \text{Transport Vehicle}(x)$?

Inheritance: $\forall x \text{ Mother}_\text{without Sons}(x) \Rightarrow \exists y \text{ Children}(x,y)$

Incoherence: $\forall x \sim \text{Grandfather}(x)$? or: ex. Interpretation $\exists x \text{ Grandfather}(x)$?

Disjunctness: $\sim \exists x \text{ Father}(x) \land \text{Mother}(x)$?

**Knowledge Base (KB):**

Consistency: does a model of the KB exist, i.e. ex. Interpretation $\mathcal{I} \models \text{KB}$?

Instantiation/Realization: Goodstrain(Z#521)?

- Find all objects $c$ with woman(c)!

- Children(Karin,Eva)?

**Formal Semantics of Terminological Logics**

**Why?**

- Specification of the meaning of representation constructs
- Comparability with other formalisms
- Algorithmization
- Correctness and complexity of inference procedures

**How?**

- Concepts refer to a set of instances: **Concept extension**
- Roles describe relations between those instances: **Role extension**
- Concept descriptions specify necessary and sufficient conditions for instances
- **Subsumption** is the necessary inclusion of concept extensions
- **Inconsistency** is the necessary emptiness of concept extensions
A Model-Theoretic Semantics for Description Logics

Interpretation function

John
Mary

Interpretation domain

Lawyer
Doctor
Vehicle

Roles

hasChild
owns

(Lawyer \ Doctor)