Exercise 1. (3 points)

Consider the clause set $\Delta = \{\neg P, Q\}, \{\neg Q, R\}, \{\neg R, P\}, \{P, Q\}, \{\neg P, \neg Q, \neg R\}\}$. Use the DPLL procedure to determine whether $\Delta$ is satisfiable or unsatisfiable. Give a complete trace of the algorithm, showing the simplified formula for each recursive call of the DPLL function, as in the lecture slides. Assume that the first call of the splitting rule chooses the proposition $P$ and assigns it the value $T$.

Exercise 2. (2 points)

Consider the clause set $\Delta = \{\neg P, Q\}, \{\neg Q, R\}, \{\neg R, P\}, \{P, Q\}, \{\neg P, \neg Q, \neg R\}\}$. Say that we run the DPLL procedure with clause learning on this formula, and the first call of the splitting rule chooses the proposition $P$ and assigns it the value $T$, i.e., the choice literal is $P@1$. Draw the implication graph, and a conflict graph. Which is the clause that we can learn, with the method presented in the lecture, from that conflict graph?

Exercise 3. (5 points)

Consider the following statements:

(i) Adam loves everybody who loves wine.

(ii) Eva or Anna love wine.

(iii) Anna does not love wine.

(iv) Adam does not love Eva.
Formalize these statements as a set $\theta$ of predicate logic formulas in prenex normal form. Then bring each formula in $\theta$ into Skolem normal form, resulting in the set of formulas $\theta^*$ (in case the formulas in $\theta$ already are in Skolem normal form, there is nothing to be done in this step). Write up the Herbrand Universe $D(\theta^*)$. Then write up the Herbrand expansion $E(\theta^*)$, which is a set of propositional formulas. Bring each of these formulas into CNF, resulting in a set $\Delta$ of clauses. Then use propositional resolution to prove that $\Delta$ is unsatisfiable. What does this tell us about the relation between Adam and Eva?